Field Map Characterization from Magnetic Survey in High Field Magnets

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Quality checks to characterize magnets before final assembly are usually foreseen when high accuracy is required in magnetic field maps. This process may possibly include magnetic field measurements in regions adjacent to the coils, thus different from the target field areas. This technique implies inverse problem approaches to estimate coils deformation parameters, and resulting accuracy may be poor due to ill conditioning of underlying mathematical processing. To counteract such drawback, suited measures are usually taken, but attention is given to estimating parameters regardless of their use. In this paper, a discussion is presented about characteristics of the inverse problem relating magnetic field and deformation parameters and about the actual accuracy needs when the interest is focused on magnetic field map in target areas rather than on parameters themselves.

***Index Terms*—High Field Magnets, Inverse Problems, Magnetostatic.**

I. Introduction

HIGH FIELD magnets share the need for both high field level and accuracy of field map in well defined regions of space (target areas). Manufacturing and assembly processes alter the design geometry of the coil, leading to a loss of accuracy. A suitable characterization of the magnet "as built", at the end of manufacturing process, can allow correcting actions or, at least, a more accurate knowledge of the actual magnet field map.

Effective information on actual layout can be estimated using the "magnetic footprint" of the magnet, which is the actual flux density map in a "measurement area", in regions as close to the magnet external surface as possible, yet easily accessible to magnetic field probes.

When the nominal geometry of the magnet and the nominal conductors distribution inside the case, i.e. the "Winding Pack" (WP) structure, are known, a reasonable approach to characterize deformed magnets is to define a limited set of "deformation parameters" describing the coil's shape, and try to "best fit" their values from measured magnet footprint. This approach requires the knowledge of the inner magnet structure, the setting of a mathematical model of such structure flexible enough to fit measurements, and an assessment of the impact of nuisances in the measurement process on the parameters' estimate. The approach is then capable, within the accuracy limits prescribed by the magnet modeling and by inverse approaches [1], to estimate the deformed shape.

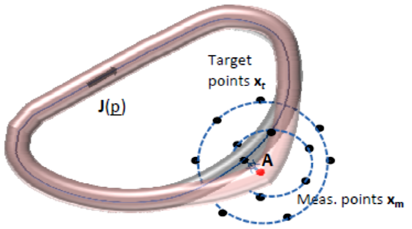

Fig. 1 – Schematic view of a magnet (grey nominal, red deformed) with shape parameter represented by the coordinates of reference point **A**; measurement points and target points are also indicated.

On the other hand, in this work the attention is focused on the representation of the field map in the target area of the actual, deformed magnets, rather than on parameters estimate. This viewpoint shift has a relevant impact on the operator relating field measurements and "output" quantities, and some relaxing on the measurement process needs is possible.

A complete discussion on operators' characteristics and the impact on the measurement process, similar to what reported in [2], will be presented in the full paper. In this short version, a synthetic mathematical model of the problem is reported in Sect. II, and an application example is presented in Sect. III.

II. Modeling of the System

The magnets characterization problem requires computing the (actual) magnet field map in a set of target points $\underline{\mathbf{x}}_t$ from measurements taken in a set of measurement points $\underline{\mathbf{x}}_m$.

Let us first introduce the direct operator $\mathcal{M}$, providing the flux density at points $\underline{\mathbf{x}}$ once the source current $\mathbf{J}(\underline{p})$ is given:

$$\underline{\mathrm{m}}\left(\underline{\mathbf{x}};\underline{p}\right)=\mathcal{M}\left(\mathbf{J}_s\left(\underline{p}\right),\underline{\mathbf{x}};\underline{s}\right) \tag{1}$$

where $\underline{p}$ is the array of parameters to be identified, characterizing the (deformed) coil, $\underline{\mathbf{x}}$ is the field points coordinates array, and $\underline{s}$ is an array characterizing the measurement process (e.g. the measurement directions). Magnetostatic law gives the structure of $\mathcal{M}$; assuming that no magnetic materials are present, $\mathcal{M}$ will be defined using integral relationships (e.g. Biot-Savart law). In the following, we will consider two "determinations" of $\mathcal{M}$: the first one, denoted as $\mathcal{M}_m$, relates $\underline{p}$ to measurements in the points $\underline{\mathbf{x}}_m$, while the second one, denoted as $\mathcal{M}_t$, relates $\underline{p}$ to measurements in $\underline{\mathbf{x}}_t$. Actually, the inverse of $\mathcal{M}_m$ will be required: the operator $\underline{p}^*=\mathcal{P}\left(\underline{m}\right)=\mathcal{M}_m^{-1}\left(\mathbf{J}_s,\underline{\mathbf{x}}_m;\underline{s}_m\right)$ maps the (measured) field onto the (estimated) parameters set $\underline{p}^*$ describing the corresponding deformed geometry. The "functional" formulation of the problem will be described with some detail in the full paper; in this digest, just the "discrete" formulation will be presented. As a matter of fact, in magnets characterization, a limited number N_p of parameters are extracted from a limited number N_m of measurements. In the hypothesis that parameters describe (very) small deformations due to manufacturing and assembly tolerances, the expression (1) can be Taylor expanded with respect to parameters variation $\delta\underline{p}$. The final expression relating the measurements $\underline{\mathrm{m}}_m$ and the deformations $\delta\underline{p}$ will then be:

$$\underline{\mathrm{m}}_m\left(\underline{\mathbf{x}}_m, \underline{p}_0+\delta\underline{p}\right) \approx \underline{\mathrm{m}}_m\left(\underline{\mathbf{x}}_m, \underline{p}_0\right)+\underline{\underline{S}}\delta\underline{p}+\underline{n} \tag{2}$$

where the reference values $\underline{p}_0$ denote nominal configuration, the sensitivity matrix $\underline{\underline{S}}$ is defined as $s_{i,j}=\left.\frac{\partial m_i}{\partial p_j}\right|_{\underline{p}_0}$, and $\underline{n}$ represents the "nuisance" on the measurements, among which just noise will be considered here. Assuming $\underline{p}_0$ is known, the inverse operator reduces to the (pseudo)-inverse of $\underline{\underline{S}}$ acting on the "differential" measurements. The characteristics of such matrix, and their relationship to the problem conditioning, will be discussed in the full paper.

The second determination of $\mathcal{M}_t$ is the operator relating (deformed) sources to field in the target area. While in many applications the inverse problem of determining deformation parameters from measurements is relevant as such, in this work we actually apply the "composite" operator:

$$\mathcal{M}_t\left(\mathbf{J}_s\left(\mathcal{P}(\underline{m}_m)\right), \underline{\mathbf{x}}_t; \underline{s}_t\right) \approx \underline{\mathrm{m}}_t\left(\underline{\mathbf{x}}_t, \underline{p}_0 + \underline{\underline{S}}^{-1}\left(\underline{\mathrm{m}}_m\left(\underline{\mathbf{x}}_m, \underline{p}\right) - \underline{\mathrm{m}}_m\left(\underline{\mathbf{x}}_m, \underline{p}_0\right)+\underline{\mathrm{n}}\right)\right) \tag{3}$$

The conditioning number of $\underline{\underline{S}}$ in (3), and therefore accuracy and reliability of solution, depend on a number of elements, including the number and position of measurement points and the number and the type of the parameters used to describe the magnet. This point will be discussed in detail in the full paper.

As a general comment, the accuracy improves by increasing the number of measurement points N_m (at least until saturation is not reached) and the number of parameters N_p used to describe the magnet. Note that ill-posedness of the matrix $\underline{\underline{S}}$ increases as well when increasing N_p. It is also worth noticing that the nearer the measurement points are, the better the identification problem conditioning gets, while the farther the target region is, the better the combined problem conditioning gets. Finally, note that under suitable hypotheses (e.g. the target area must be further than the measurement area, thus relating (3) to a well-posed exterior Cauchy problem), the application of $\mathcal{M}_t$ regularizes the problem, as will be shown in the following section. A more thoughtful description, similar to what presented in [2], [3], will be given in the full paper.

III. Example of Application

The characterization of a Toroidal Field Coil (TFC) for next generation ITER Tokamak [4] is presented here as an example. TFCs are rather large, but must be manufactured within few millimetres tolerance to guarantee desired performance. A 12.65 m high TFC, with the same shape as in Fig, 1, and wound using 130 turns of series connected conductor is considered. Deformation parameters for TFCs are typically represented by in-plane and perpendicular shifts of a few "control points" along the geometrical current centreline of the magnet (the black line in Fig. 1). The deformation considered here is a 5 mm perpendicular and 3 mm planar shift of point **A**, while in the full paper more complex deformations will be considered. In order to clarify the relevant aspects of the method, the flux density is measured just along a circumference with 0.3 m radius (smallest achievable distance due to the coil structure), centred in **A**, using 12 3D probes. In the full paper, the positive impact, in terms of accuracy and robustness, of increasing the number of measurements will be demonstrated. The target area is a circumference with a radius of 0.6 m (see Fig. 1). Additive Gaussian noise is considered, with 1% std. deviation. Three models are used to characterize the TFC from near field measurements, namely:

a. A model coherent with the actual WP, and able to deform both on the coil plane and perpendicularly;
b. Same as *a*, except that only perpendicular deformations are considered;
c. A model with 18 equivalent conductors in the WP, able to deform both on the coil plane and perpendicularly to it.

In all cases, the TFC shape is defined by its current centreline, while WP structure is kept in the deformations.

TABLE I
CONDITIONING NUMBER AND RECONSTRUCTION ERRORS

Case	Cond. Number of $\underline{\underline{S}}$	Measurement error in target area (%)	Reconstruction error on parameters (%)
a	11.8×10^4	0.8%	7%
b	0.5×10^4	2%	29%
c	3.0×10^4	4%	130%

In Table I the conditioning number of $\underline{\underline{S}}$, the average error on measurements in the target area and the average parameters reconstruction error are reported. Results show that:

– The 18 filaments model *c* is not able to reconstruct the coil deformation parameters, which was to be expected, but it is able to reconstruct the field map in the target region with satisfactory accuracy;

– The reduction of N_p improves the problem conditioning, but decreases the representation capability and the accuracy.

IV. Conclusions

The magnet characterization problem has been studied, taking advantage of a decomposition in two sub problems, which allows identifying the ill-posed nature of the "source identification" sub-problem, but also the regularizing nature of the "mapping" sub-problem. An illustrative example was presented, but a broader discussion will be given in the full paper, where a methodology similar to the approach usually adopted for current profile reconstruction will be fully presented.

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References

[1] V. Isakov, *Inverse Source Problems*, Providence: American Mathematical Society, 1985.

[2] A. El Badia, T. Ha Duong, "Some remarks on the problem of source identification from boundary measurements", *Inverse Problems* (1998), Vol. 14, n. 4, pp. 883-891, 1998.

[3] A. Bonito Oliva et al, "Equivalent current models for magnets characterization", *Proc. of IGTE Symposium 2014*, Graz (Au), Sept. 2014.

[4] A. Bonito Oliva et al, "Characterization of deformed magnets from external magnetic measurements", *IEEE Trans. on Magnetics*, 50 (2), DOI: 10.1109/TMAG.2013.2282315.